Reply to "Comment on 'Boundary conditions in the Unruh problem' "

N.B. Narozhny, A.M. Fedotov, B.M. Karnakov, and V.D. Mur
Moscow Engineering Physics Institute, 115409 Moscow, Russia

V.A. Belinskii
INFN and ICRA, Rome University "La Sapienza," 00185 Rome, Italy
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We reply to the preceding Comment by Fulling and Unruh criticizing our conclusion that principles of quantum field theory as of now do not give convincing arguments in favor of a universal thermal response of detectors uniformly accelerated in Minkowski space [Phys. Rev. D **65**, 025004 (2002)]. We maintain our conclusion and present additional arguments to confirm it.

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I. INTRODUCTION

We concluded in our paper [1] that "principles of quantum field theory as of now do not give convincing arguments in favor of a universal thermal response of detectors uniformly accelerated in Minkowski space." To be more precise, we showed that the Unruh procedure [2] was an invalid quantization scheme in Minkowski spacetime (MS). Hence this procedure cannot introduce the notion of Fulling-Rindler (FR) particles [3] in MS, so that the assertion that the Minkowski vacuum is a thermal state of FR particles loses any sense.

The authors of the Comment [4] on our work argue that it deals mainly with "mathematical technicalities" irrelevant to the "Unruh effect," and thus our conclusions are "unwarranted." We disagree with this conclusion, and in this Reply we will try to clarify the issues touched on by the authors of the Comment. For the reader's convenience the structure of our notes basically repeats the structure of Ref. [4]. We use the same notation as in Refs. [1,4].

II. BOUNDARY CONDITIONS IN THE UNRUH PROBLEM

A. Rindler and Minkowski representations and boundary (decay) conditions

The conclusion of Sec. II A of Ref. [4] reads: "Any mathematical pathologies encountered in attempting to extend the FR representation of the field algebra right onto the horizon are not relevant, either to Unruh's work (because it does not deal with the FR representation) or to Fulling's (because the horizon and the regions beyond it are not part of the spacetime considered there)."

Let us first comment on Fulling's work [3]. We have never called in question the consistency of Fulling quantization in Rindler spacetime (RS) regarded as a self-contained, globally hyperbolic, static spacetime. But we analyzed Fulling's procedure in Sec. III of our paper [1] and showed that it implies zero boundary (decay) condition at space infinity of RS. The authors of Ref. [4] agree that this boundary condition [of type (iii) according to Ref. [4]] exists, but assert that it is "totally irrelevant to what happens in R, identified with RS." We cannot agree with this assertion. The discussed boundary condition is just the standard restriction for any physical field to decay at spatial infinity and is of crucial importance for the structure of the quantum field in inner points of the corresponding spacetime, say RS. This follows from the fact that operator coefficients in expansion of the field in terms of a complete set of orthonormalized modes acquire the meaning of creation and destruction operators under at least two requirements. First, they must obey canonical commutation relations; and second, any physical quantities associated with these operators, such as, e.g., the energy of a one-particle state, should be finite. Our boundary condition just ensures the satisfaction of the second requirement. Moreover, it was shown in Ref. [1] that this boundary condition follows from the finiteness of one-particle states with necessity. This agrees with the result of Parentani, who showed in Ref. [5] that the average energy of a Fulling-Rindler particle in MS is infinite.

The second requirement usually goes without saying and is not discussed in textbooks, as a rule. It would not be worth discussing it in connection with the Fulling quantization either, if the work [3] dealt with a field located exclusively in RS. However, the author of Ref. [3] makes an attempt to compare the notion of FR particles with the standard particle concept in MS. This is done by expanding the Minkowski field restricted to the R wedge of MS in terms of Fulling modes. This mathematically legal operation results in Eq. (30) of Ref. [3] and is interpreted as a relation between creation and destruction operators for FR and standard MS particles. However, the restriction of the Minkowski field to RS *does not obey* the zero boundary condition. Hence the operator a_j in the left-hand side of Eq. (30) does not satisfy the second requirement and thus cannot be identified with the destruction operator of a FR particle introduced in Sec. II A of Ref. [3]. Therefore the absolutely correct conclusion of Ref. [3] that "The notion of a *particle* is completely different in the two theories" arises not so much from the nonvanishing kernel $V(j,k)$ in Eq. (30) of Ref. [3], as from the fact that these two theories arise in two absolutely different physical problems. One of these problems is quantization of the field in global MS with zero boundary conditions at space infinities, $z\rightarrow\pm\infty$, z is the space coordinate in MS, while the other is the quantization in RS with zero boundary condition at $\rho\rightarrow 0$ and $\rho\rightarrow\infty$, where ρ is the RS space coordinate. The field quantized in RS according to the Fulling prescription and the field quantized in MS with the conventional procedure *cannot be identified* even in the R wedge of MS.[1]

Let us emphasize that the author of Ref. [3] has considered the possibility of this interpretation. It is written in Ref. [3], p. 2856 that "the a quantization in Rindler space [the Rindler-Fock representation, Eq. (23)] can be interpreted as that appropriate to the physical situation of an impenetrable wall located on the light cone $z=0$, or, more realistically, to the limiting case of a wall which accelerates along one of the curves $z=z_0$, where z_0 is a small positive constant," see also Ref. [6]. According to Ref. [3] this interpretation implies imposing the "perfect reflection" boundary condition[2] that Fulling modes are equal to zero at the wall (on the light cone). Our interpretation almost literally coincide with that one. There are only two distinctions: (i) we impose the zero boundary condition not for the modes (they are already distributionally equal to zero at the point $\rho=0$), but for the field operator in a weak sense (see Ref. [1]); and (ii) our point is that particle interpretation of the quantum field in RS is possible only if the field operator obeys the zero boundary condition. It is clear that in the framework of this interpretation FR representation is not relevant to the "Unruh effect," so on that point we agree with the authors of Ref. [4].

We also agree that any attempt to extend the FR vacuum as a possible state of the field throughout MS is a "dubious enterprise." However, such an attempt was undertaken in the Unruh work [2]. To make sure of this one should look at Eqs. (2.10)–(2.12) of Ref. [2]. In particular, Eq. (2.12) of Ref. [2] is the union of the field expansions in FR modes for the R and L wedges of MS. Then the right and left FR modes are "analytically extended across the future and past horizons into the regions F and P of MS" to make "the expansion of Eq. (2.12) valid in the full MS;" see Ref. [2], p. 879. It is worth noting that the operator coefficients of the expansion (2.12) are exactly creation and destruction operators of FR particles for R and L wedges of MS and define the right and left FR vacua already as a state of the field in the full MS [see Eq. (2.11) and the unnumbered formula following Eq. (2.19a) of Ref. [2]]. It is written in Ref. [2, p. 880] that "the Minkowski vacuum can be reexpressed as a many-particle FR state." Then "by comparing the expansion of the field operator in terms of the Minkowski positive-frequency basis, and comparing the resultant creation and annihilation operators with the FR set" the author arrives at Eqs. (2.19a), (2.19b) of Ref. [2]. Thus we conclude that the assertion of the authors of the Comment [4] that the "Unruh work does not deal with the FR representation" contradicts the actual content of Ref. [2].

It must be clear from the above discussion that the notion of FR particles implies the existence of a zero boundary condition for the field operator at space infinity of RS embedded in MS as the region R. It is clear also that the field operator in MS does not obey that restriction. However, by virtue of the fact that the space infinity of RS is mapped to a single point of MS, namely, the origin $x=0$, the authors of Ref. [4] argue that "the subtlety about the origin" does not affect the physics in $R\cup L$. We will discuss this point in the succeeding section.

B. The zero boost mode

We agree with the authors of the Comment that singularity of the boost mode, as a function of κ, at the origin of MS is "not relevant to the behavior of an eigenfunction expansion at points inside the wedges R and L." However, this statement, in contrast to Ref. [3] (at least to that part of it which deals with RS as a self-contained spacetime), is completely irrelevant to Ref. [2] which attempts to consider the field states defined *throughout* MS.

The fact that the data at a single point of a Cauchy surface can play the decisive role for dynamics of the system in MS is confirmed by the Cauchy data for the two-point commutator in both the four- (the Pauli-Jordan function; see, e.g., Ref. [7]), and the two-dimensional cases [see Eq. (2.16) of Ref. [1]]. This function is equal to zero throughout the surface $t=0$, while its time derivative is a delta function of $\mathbf{r}$, i.e., is not equal to zero only at the origin. However, it is claimed in Ref. [2], p. 879 that "the union of the two spacelike hypersurfaces given by $\tau=0$ in both R and L is a Cauchy surface for the full Minkowski spacetime," where τ is the Rindler time. Since R and L are *open* regions, this union reproduces the surface $t=0$ without the origin. The above mentioned example shows that such a surface *cannot* serve as a Cauchy surface in full MS. Otherwise the two-point commutator in the theory (which is Poincaré invariant) would be identically equal to zero throughout MS including the points inside the light cone. That would mean that there is no quantum theory in MS with the excised origin. Naturally, it is senseless to talk about the Minkowski vacuum in such a theory and, as a consequence, about the thermal properties of this state from the point of view of any observer. It is worth noting that Troost and Van Dam came to exactly the same conclusion. They explicitly demonstrated in Ref. [8] that the thermal properties of the vacuum propagator are straightforwardly related to excising the origin from the Euclidian spacetime obtained from MS by Wick rotation. In MS this procedure is equivalent to cutting out the horizons, in complete agreement with our result.

We showed in Ref. [1] that cutting out the origin from the surface $t=0$ is equivalent to omitting the zero boost mode in expansion of the field operator in MS in terms of boost modes, Eq. (4.6) of Ref. [1]. However, the authors of Ref. [4] assert that "the omission of the single point $\kappa=0$ will not change the value of the integral" (5.8) in Ref. [1]. It is said that this is because "the natural measure for integration over κ [in Eq. (5.8) of Ref. [1]] is the ordinary Lebesgue measure." This assertion is faulty because, by virtue of the δ-function type singularity of the zero boost mode *on horizons and at the origin* [see Eq. (4.19) in [1]], the spectral point $\kappa=0$ is of *nonzero* measure *there* (Dirac measure; see

[1] It is worth noting that, if it were not so, Eq. (30) of Ref. [3] could serve the definition of Fulling operators a_j "throughout" MS just in the sense of the joke of the authors of Ref. [4] about Boston post office.

[2] In this context we do not quite understand the criticism in Ref. [4] of our terminology.

Ref. [9], note 1.4). Thus, being indeed Lebesgue negligible inside R and L, this spectral point *gives an essential contribution* to expansion of the field *in the whole MS* Ref. [1], Eq. (4.6).

The authors illustrate their point by an example with the eigenfunction expansion of a *continuous* function $g(x)$. We agree that the relation

$$g(x)=\lim_{\epsilon\to 0}\int_{|x'|>\epsilon}\delta(x-x')g(x')dx' \qquad (1)$$

understood as a distributional equality is correct and that the zero boost mode does not contribute to the right hand side of Eq. (1). However, one does not deal only with continuous functions in quantum field theory. Let us consider the standard singular Wightman two-point function as an alternative example.

Using the formulas of Ref. [1], Sec. IV the following representation for the Wightman function $\Delta^{(+)}(x-x';m)$ of the massive scalar field in two-dimensional MS can be derived:

$$\Delta^{(+)}(x-x';m)=i\langle 0_M|\phi_M(x)\phi_M(x')|0_M\rangle = i\int_{-\infty}^{\infty}d\kappa\Psi_\kappa(x)\Psi_\kappa^*(x'). \qquad (2)$$

By virtue of the translation invariance of the Minkowski vacuum we arrive at

$$\Delta^{(+)}(x-x';m)=i\int_{-\infty}^{\infty}d\kappa\Psi_\kappa(x-x')\Psi_\kappa^*(0). \qquad (3)$$

Note that the representation (3) holds for arbitrary points x, x' but not only for the case when one of the points is fixed at the origin. Let us now omit the spectral point $\kappa=0$ in the integral in the right hand side of Eq. (3), i.e., make the change

$$\int_{-\infty}^{\infty}d\kappa\cdots\to\lim_{\epsilon\to 0}\int_{|\kappa|>\epsilon}d\kappa\cdots. \qquad (4)$$

It is easily seen that since $\Psi_\kappa(0)=1/\sqrt{2}\,\delta(\kappa)$ [see Eq. (4.17) in [1]] the right-hand side of Eq. (3) turns into zero after the change (4), and by virtue of Wightman's reconstruction theorem [10] the Minkowski vacuum state also becomes equal to zero.

This is an additional argument in favor of the inadmissability of omission of the zero boost mode in the field expansion in terms of boost modes in MS.

C. Local observables and the Bisogniano-Wichmann theorem

It is stated in [4] that ''the thermal nature of the state (Minkowski vacuum) can alternatively be derived without even mentioning these (boost) modes;'' see also, e.g., [11,12]. It is considered that the most consistent approach for derivation of the Unruh effect, which appeals neither to the Unruh modes and the Fulling quantization scheme, nor to concrete models of detectors, is the approach based on the Bisogniano-Wichmann (BW) theorem in the framework of an algebraic formulation of quantum field theory. However, we have shown that, based on the correct mathematical theorem (the BW theorem), this approach encounters the same difficulties as the conventional one.

The authors of the Comment disagree with our conclusion. They are right to state that, because of Eq. (2) of Ref. [4] [Eq. (6.31) in [1]], the restriction of the Minkowski vacuum to observables localized in the R (or L) wedge of MS coincides with the thermal state[3] relative to the ''boost Hamiltonian,'' but they identify this statement with the Unruh effect. This is not the case however. The point is that the set of observables which can be measured by a right or left Rindler observer in MS cannot be reduced to the elements of the algebra $\tilde{\mathcal{U}}$ of observables localized in $R\cup L$, since the complete set also includes the observables that ''catch on'' the origin of MS. The Unruh effect, as a physical statement, in the framework of the algebraic approach would imply that the KMS condition was satisfied with respect to the *complete* set of observables accessible for Rindler observers. However, the BW theorem holds only on $\tilde{\mathcal{U}}$ and any continuation of the functional $\tilde{\omega}_F^{(2\pi)}$ from Eq. (2) of [4] does not obey the KMS condition. This situation is in full analogy with the conventional approach which we discussed above. Equation (2) of [4] holds only on the algebra $\tilde{\mathcal{U}}$, which *is not the complete set* of observables accessible to Rindler observers since the ''catching on'' observables are excluded, while the Unruh quantization is valid only in $R\cup L$ after excluding the zero mode from the set of boost modes complete in MS. An explicit example of a ''catching on'' observable is presented in the Appendix.

Certainly, a physical phenomenon can be studied using different mathematical tools but we believe that the nature of the phenomenon will be the same independently of the method of investigation. The method of expansion of the field operator in terms of a complete orthonormalized set of modes, as well as the particle language based on this method, are commonly accepted in quantum field theory. We are sure that, if some inconsistency arises while treating by this method a free field even in a noninertial reference frame, it will persist in the framework of any other appropriate method of investigation. We think also that any other method including one that ''does not even mention'' boost modes will necessarily employ the notion of FR particles by this way or another. This is because if one manifests the presence of a thermal bath in MS one should indicate which concrete medium is heated to the Davies-Unruh temperature. Remarkably, the authors of all papers and monographs known to us including the original work [2] interpret the thermal property of the Minkowski vacuum in terms of FR particles. This statement refers, in particular, to Refs. [13] [see Eqs. (11), (12) therein] and [11], in which the approach employing the Minkowski vacuum two-point function is used. It is worth noting that in the framework of that approach one expresses

[3]The term ''Kubo-Martin-Schwinger (KMS) state'' would be more precise.

a *detector response* function through the vacuum two-point function on the trajectory of a uniformly accelerating detector (compare Refs. [11,13]). But this means that it is necessary to employ a concrete model of detector using this approach; see Sec. III C below for further discussion.

III. OTHER ISSUES

In this section we will comment on different questions raised by the authors of Ref. [4] which need clarification although they are not of the utmost importance for the subject of the present discussion.

A. Correlations and the superselection rule

We used the term ‘‘superselection rule’’ with only the purpose of indicating that it is impossible to prepare a state with quantum correlations between right and left FR particles in the double RS $R\cup L$. We should emphasize: *in double RS, not in MS*. This is because we have shown that the Unruh quantization scheme is valid only in double RS, which does not include the F and P regions. Quantum correlations between R and L regions simply cannot be prepared in this spacetime.

Consider the Wightman two-point function of the field ϕ_{DW} in the double RS

$$\Delta^{(+)}_{DW}(x,x';m)=i\langle 0_{DW}|\phi_{DW}(x)\phi_{DW}(x')|0_{DW}\rangle = i\int_0^\infty d\mu\{R_\mu(x)R^*_\mu(x')+L_\mu(x)L^*_\mu(x')\}, \tag{5}$$

where $R(x)$ and $L(x)$ are the right and left Unruh modes, and $|0_{DW}\rangle$ is the vacuum state in the double RS which is annihilated by destruction operators for right and left FR particles (see Ref. [1]). It is easily seen from Eq. (5) that $\Delta^{(+)}_{DW}(x,x';m)=0$ if the points x and x' belong to different wedges of double RS. This fact in no way contradicts the well known possibility of quantum correlations between the R and L regions of MS which certainly can be created by some cause in the P region of MS. The existence of such correlations is demonstrated by a nonvanishing Wightman function in MS with two points located one in the R and the other in the L region.

Since the dynamics of the fields in the R and L wedges of the double RS (governed by boost translations) is completely independent, and communications between these wedges are forbidden by relativistic causality, it is impossible starting from the vacuum in double RS to create correlations between the left and right particles in this spacetime.

B. State preparation

Commenting on our statement that a uniformly accelerated observer cannot prepare the Minkowski vacuum state as the initial state of the field, the authors of [4] suggest that ‘‘No observer can prepare an absolute vacuum throughout all space,’’ which is true but has no relation to our arguments. It is clear that a single observer cannot prepare any state in any spacetime simply because the quantum state is a global characteristic of the field. Thus, a quantum state of a field can be prepared only by the efforts of all the observers located throughout the space. The word ‘‘throughout’’ is understood here in the sense that the necessary initial state can be prepared with arbitrary accuracy required by a concrete experiment in a concrete physical situation. However, the Rindler observers have no access to the left wedge, and thus cannot prepare the vacuum state throughout all Minkowski space. That is what we meant in the criticized statement. In this context the very formulation of the problem about a Rindler observer *moving in a Minkowski vacuum* seems to us senseless. The Rindler observer simply cannot possess information about the state of the field in the full MS.

C. Detectors

It is stated in Ref. [4] that ‘‘the reality of acceleration temperature has been confirmed by various theoretical analysis of concrete systems.’’ However, we never asserted that a particular uniformly accelerated detector cannot reveal an Unruh (thermal-like) response. Rather, we questioned whether this response is universal, so that it could be considered as a property of the Minkowski vacuum seen from a uniformly accelerating reference frame. Certainly, the response of a particular detector can be predicted in principle by performing calculations using the standard quantum mechanical technique in an inertial reference frame without any reference to Unruh modes or the notion of FR particles. This could be done if one knows the nature of the accelerating force and the concrete structure of the detector. However, there is no hope of proving universality of the Unruh effect from an arbitrary number of detector models. That is why the examples given in Sec. III of Ref. [4] cannot convince us of the existence of the Unruh effect as a *universal* property of the Minkowski vacuum. Nevertheless, nonuniversality of the thermal response could be proven within such an approach just by demonstration of at least a single example of a uniformly accelerated detector which does not reveal the Unruh behavior. We found such an example in Ref. [14]. It was shown there that the long time behavior of an elementary particle detector accelerated by a scalar background in the vacuum of a fermion field crucially differs from the behavior one would predict on the basis of the Unruh conjecture. Instead of arriving at some ‘‘thermal equilibrium’’ state, the detector creates a neutrino-antineutrino pair and returns to its initial state. This means that the detector remembers its initial state and hence one can by no means talk about equilibrium in the final state of this particular case of a detector. We think that this example demonstrates nonuniversality of the Unruh response and constitutes a convincing argument in favor of the impossibility of proving the existence of a universal Unruh effect in the framework of quantum field theory.

IV. CONCLUSIONS

Proper understanding of the physics in accelerating reference frames is important for constructing quantum field theory in an arbitrary strong gravitational field. The basic conclusion of Ref. [1] is that the work [2] (as well as other work on the subject) does not give compelling grounds as of now for treating the Minkowski vacuum state as a thermal bath of FR particles. We hope that the additional arguments presented in this Reply will help to clarify a certain misunderstanding around our results which could have arisen through our fault, maybe because our wording sometimes was not exact enough. We hope that our observations will be useful for construction of a consistent theory.

V. FINAL REMARK

In this section we will reply to the comment in the last paragraph in Sec. III of Ref. [4] which appeared after the authors of Ref. [4] became familiar with the text of the present paper. The authors contend that ‘‘one need never use (2.12) (from Ref. [2]) in Minkowski space except with smooth test functions’’ and that omission of one spectral point in the expansion of a smooth function of compact support in terms of boost eigenfunctions does not affect the value of the integral. We have no objections to the first part of this statement but cannot agree with its second part if it applies to the point $\kappa=0$. We have written already in Sec. I B that the spectral point $\kappa=0$, due to singularity of the zero boost mode, is a point of *Dirac* but not *zero* measure on the light cone. Here we will explain this issue in detail.

The expansion of a field operator in boost eigenfunctions with excised point $\kappa=0$, which is equivalent to Eq. (2.12) of Ref. [2], reads

$$\phi(x)=\lim_{\epsilon\to 0}\phi_\epsilon(x),$$

$$\phi_\epsilon(x)=\int_{|\kappa|>\epsilon}d\kappa\{b_\kappa\Psi_\kappa(x)+b_\kappa^\dagger\Psi_\kappa^*(x)\}. \tag{6}$$

The assertion that the expansion (6) is valid in the full Minkowski spacetime means that the operator-valued distribution $\phi_\epsilon(x)$ tends to $\phi_M(x)$ as $\epsilon\to 0$, where

$$\phi_M(x)=\int_{-\infty}^{\infty}d\kappa\{b_\kappa\Psi_\kappa(x)+b_\kappa^\dagger\Psi_\kappa^*(x)\},$$

is the operator of the massive scalar field in MS.

Consider now the space of normalized Gaussian test functions

$$f^R(x)=\frac{1}{\pi R^2}\exp\left(-\frac{t^2+z^2}{R^2}\right)$$

(see the Appendix). The operator $\phi_\epsilon(x)$ smeared with a test function from this space reads

$$\phi_\epsilon(f^R)=\int_{|\kappa|>\epsilon}d\kappa f_\kappa(R)\{b_\kappa+b_\kappa^\dagger\},$$

where

$$f_\kappa(R)=\frac{1}{2^{3/2}\pi}K_{i\kappa/2}\left(\frac{m^2R^2}{4}\right) \tag{7}$$

(see the Appendix). The function $f_\kappa(R)$ is a bell-like function of κ at fixed R, and as $R\to 0$ turns into a δ function [17]

$$f_\kappa(0)=\frac{1}{\sqrt{2}}\delta(\kappa).$$

It is clear that there exists such an R_ϵ that the widths of the test functions $f_\kappa(R)$ [Eq. (7)] with $R<R_\epsilon$ become less than ϵ. Also, we can always choose a normalized state $|g\rangle=\int_{-\infty}^{+\infty}d\kappa g(\kappa)b_\kappa^\dagger|0_M\rangle$ where $g(\kappa)$ is concentrated in the interval $-\epsilon<\kappa<\epsilon$. Then the matrix element $\langle g|\phi_\epsilon(f^R)|0_M\rangle$ evidently vanishes. At the same time, due to the unboundedness of the smeared field operators, the state $|g\rangle$ can be chosen so that the matrix element $\langle g|\phi_M(f^R)|0_M\rangle$ will be arbitrarily large. Hence, there exists such a $\lambda>0$ that for any $\epsilon>0$ one can find test functions and normalized one-particle states $|g\rangle$ such that

$$|\langle g|[\phi_\epsilon(f^R)-\phi_M(f^R)]|0_M\rangle|>\lambda. \tag{8}$$

This means that the operator-valued distribution $\phi_\epsilon(f^R)$ does not converge to $\phi_M(f^R)$ when $\epsilon\to 0$ even in a weak sense.

We will illustrate the proposition by direct calculation of the limits of the discussed one-particle amplitudes for $R\to 0$. Evidently,

$$\lim_{R\to 0}\langle g|\phi_\epsilon(f^R)|0_M\rangle=\int_{|\kappa|>\epsilon}d\kappa\frac{\delta(\kappa)}{\sqrt{2}}g(\kappa)=0.$$

At the same time

$$\lim_{R\to 0}\langle g|\phi_M(f^R)|0_M\rangle=\int_{-\infty}^{\infty}d\kappa\frac{\delta(\kappa)}{\sqrt{2}}g(\kappa)=\frac{g(0)}{\sqrt{2}}.$$

Let us emphasize that the latter formulas were obtained in our paper [1], Eqs. (5.15), (5.10), respectively, where they were written for field operators and had to be understood in the weak sense. Here they are derived for smeared operators.

Let us give another example illustrating the fact that excising the point $\kappa=0$ strongly affects the properties of the field operators. Consider the matrix element of the operator $\phi(f)$, smeared with an arbitrary test function, between a non-normalizable one-particle state with $\kappa=0$ and the vacuum state in MS,

$$\langle 0_M|b_0\phi(f)|0_M\rangle=\lim_{\epsilon\to 0}\int_{|\kappa|>\epsilon}d\kappa f_\kappa\delta(\kappa).$$

It is clear that the considered matrix element is equal to zero while the matrix element $\langle 0_M|b_0\phi_M(f)|0_M\rangle$ is evidently equal to $f_0\neq 0$.

Thus we see that the field $\phi(x)$, Eq. (6), has nothing to do with the quantum scalar massive field $\phi_M(x)$ in MS. The reason for this result is very simple. To construct a quantum field theory one should have field operators smeared with all possible smooth test functions. But in that functional space test functions always exist with arbitrarily small dispersion localized at the light cone where the zero boost mode has a δ-function singularity. That is why omission of the zero boost mode is far from being a harmless operation. We hope that after this clarification it will be clear that excising the singular zero mode out of the set of boost eigenfunctions is inadmissible.

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APPENDIX: THE "CATCHING ON" OBSERVABLES IN THE FIELD ALGEBRA IN MS

The BW theorem states that the Minkowski vacuum Green's functions, being restricted to the right (or left) Rindler wedge of MS, satisfy the KMS condition with dimensionless "temperature" $(2\pi)^{-1}$ relative to the Lorentz boost generator $\hat{L}$; see, e.g., [15]. As an example, consider the particular case of the two-point Wightman function in MS. Denote by $\phi_M(f)$ a field operator smeared with a smooth test function $f(x)$, $\phi_M(f)=\int dx\,\phi_M(x)f(x)$. If $\mathrm{supp}\{f\}\in R$, then the BW theorem is equivalent to the assertion [15] that the state

$$|\Theta_f(\eta)\rangle=e^{i\eta\hat{L}}\phi_M(f)e^{-i\eta\hat{L}}|0_M\rangle=e^{i\eta\hat{L}}\phi_M(f)|0_M\rangle$$

is an analytical function inside the strip $0<\mathrm{Im}\,\eta<\pi$ and

$$\langle\Theta_g(i\pi)|\Theta_f(i\pi)\rangle=\langle\Theta_f(0)|\Theta_g(0)\rangle.$$

Now consider a test function which is localized near the origin,

$$f(x)=C\exp\left(-\frac{t^2+z^2}{R^2}\right),$$

where R and C are constants. Since $\mathrm{supp}\{f\}\cap R\neq\varnothing$, this test function defines a "catching on" observable. We have

$$|\Theta_f(\eta)\rangle=\int_{-\infty}^{+\infty}d\kappa f_\kappa e^{i\kappa\eta}b_\kappa^\dagger|0_M\rangle,\quad f_\kappa=\int dx\Psi_\kappa^*(x)f(x),$$

where b_κ and Ψ_κ are the boost destruction operator and the boost mode, respectively. Making use of the boost mode integral representation, Eq. (4.7) in [1], we obtain

$$f_\kappa=\frac{CR^2}{2\sqrt{2}}K_{i\kappa/2}\left(\frac{m^2R^2}{4}\right),$$

where $K_{i\kappa/2}(y)$ is a Macdonald function.

In order for the state $|\Theta_f(\eta)\rangle$ to be an analytical function inside the strip $0<\mathrm{Im}\,\eta<\pi$, it is necessary that all the overlaps $F(\eta)=\langle\chi|\Theta_f(\eta)\rangle$ are also analytical functions inside that strip. It is assumed that the states $|\chi\rangle$ are normalized, e.g., by the condition $\langle\chi|\chi\rangle=1$. For the state

$$|\chi\rangle=\frac{1}{\sqrt{2\pi K_0(m^2R^2/2)}}\int_{-\infty}^{+\infty}d\kappa K_{i\kappa/2}\left(\frac{m^2R^2}{4}\right)b_\kappa^\dagger|0_M\rangle,$$

the overlap can be calculated analytically [16],

$$F(\eta)=\frac{CR^2}{2}\sqrt{\frac{\pi}{K_0(m^2R^2/2)}}K_0\left(\frac{m^2R^2}{2}\cosh\eta\right).$$

The function $F(\eta)$ possesses branch points at $\eta=i\pi(n+1/2)$, $n=0,\pm1,\pm2,\ldots$. Therefore the state $|\Theta_f(\eta)\rangle$ is free of singularities only inside the strip $-\pi/2<\mathrm{Im}\,\eta<+\pi/2$, and hence the KMS property for the Wightman function fails. Moreover, the KMS condition *cannot even be formulated* for the "catching on" observables. Certainly, this result by no means contradicts the BW theorem.

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